

ALDE 봄학기 연구계획 발표 (COPS & ROBBER)

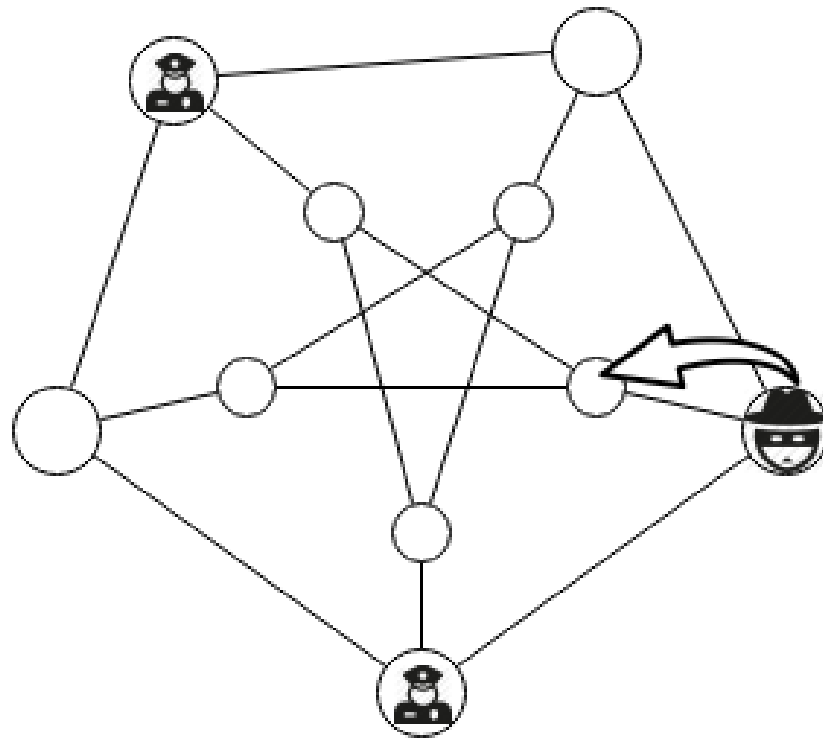
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1. COPS & ROBBER

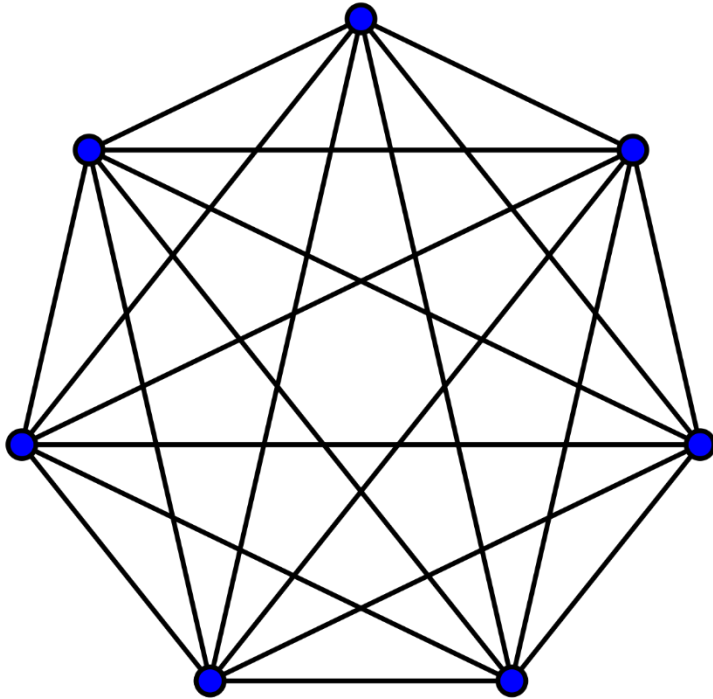
- 경찰과 도둑 게임
 - 경찰은 도둑을 잡아야 승리
 - 도둑은 경찰에게 영원히 안 잡힐 수 있으면 승리(=cycle이 발생할 경우 승리)
- 여러가지 변형 조건
 - 경찰의 수, 도둑의 수
 - Map : Planar graph, Grid, Cylinder, ...
 - Movement : Fast-Robber, Lazy-Cop, ...
 - Time : Turn-based / Concurrent movement, ...



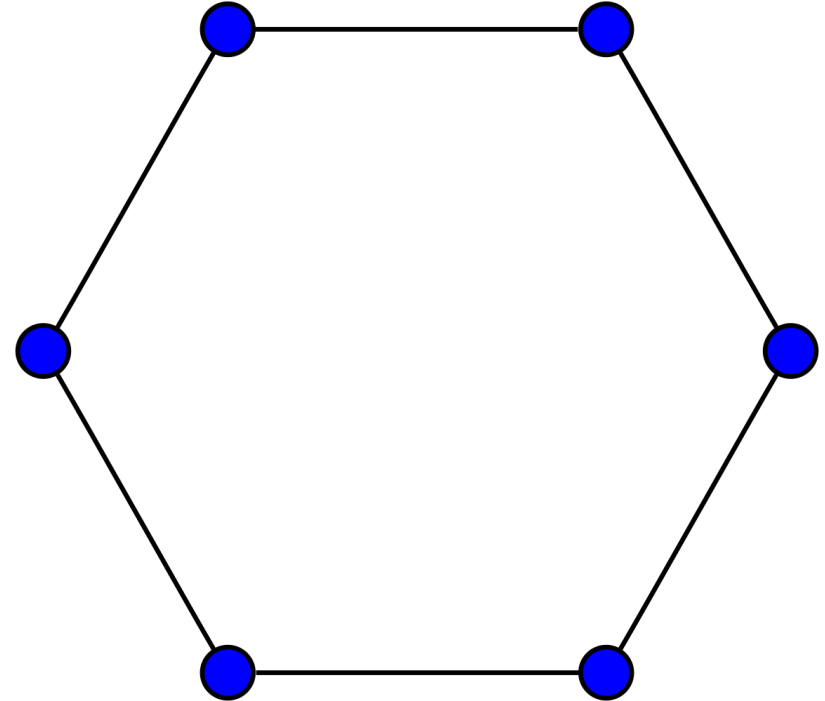
2-1. Backgrounds



2-2. Type of Graphs

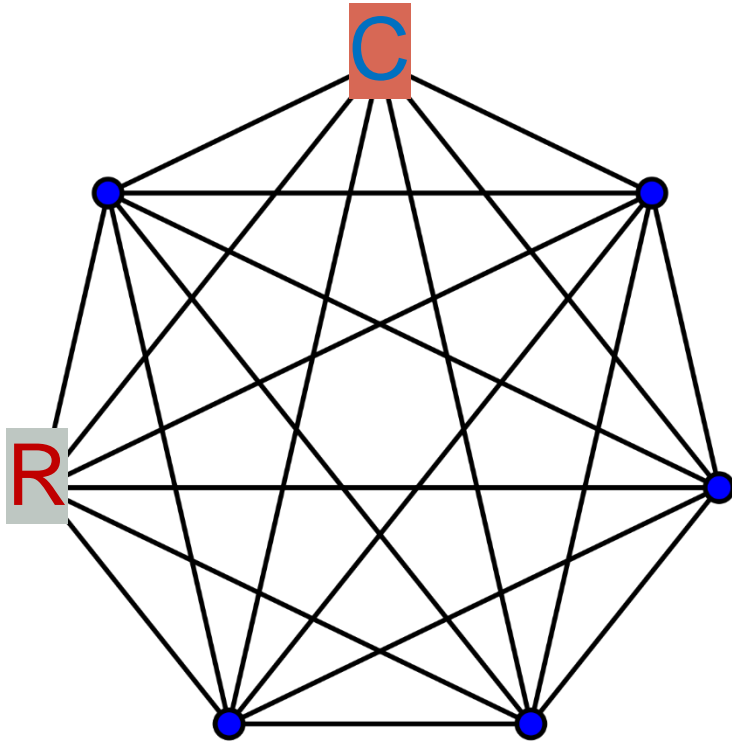


Cop-win graph

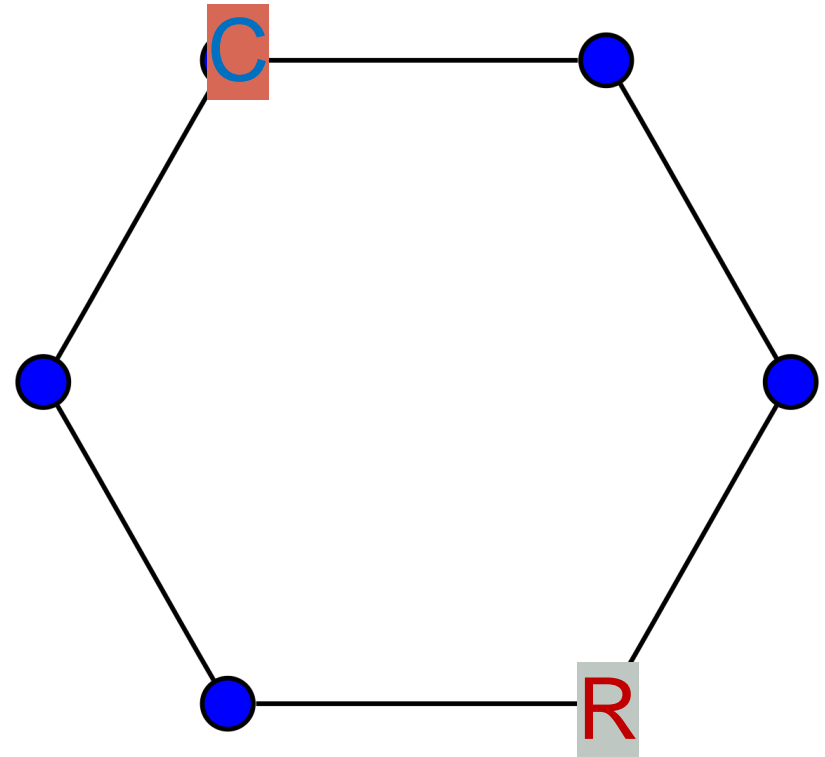


Rob-win graph

2-2. Type of Graphs

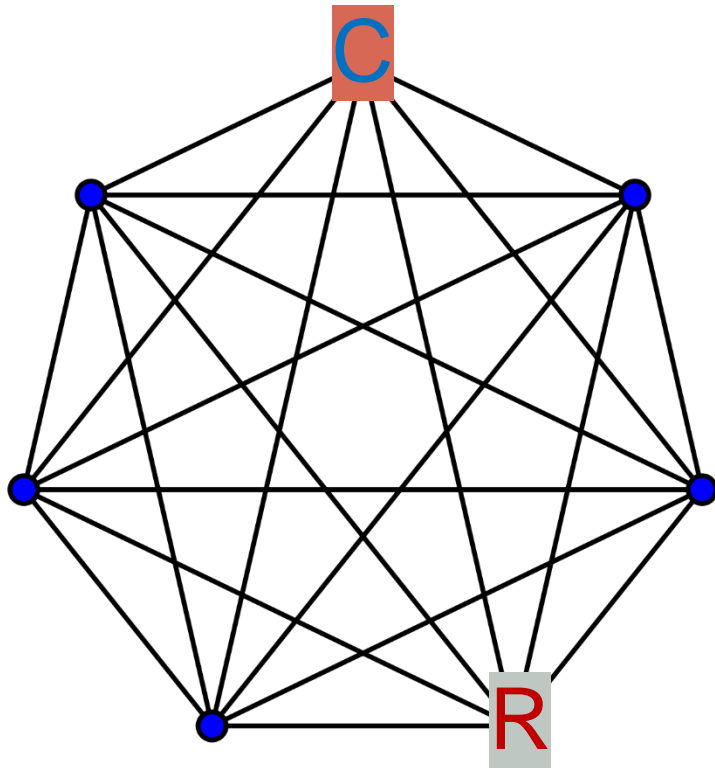


Cop-win graph

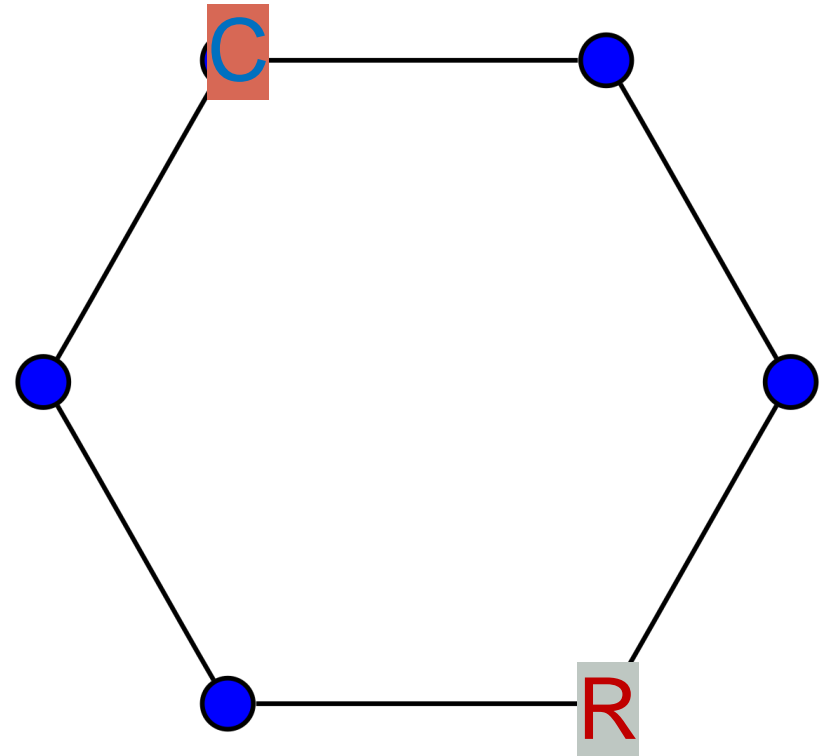


Rob-win graph

2-2. Type of Graphs

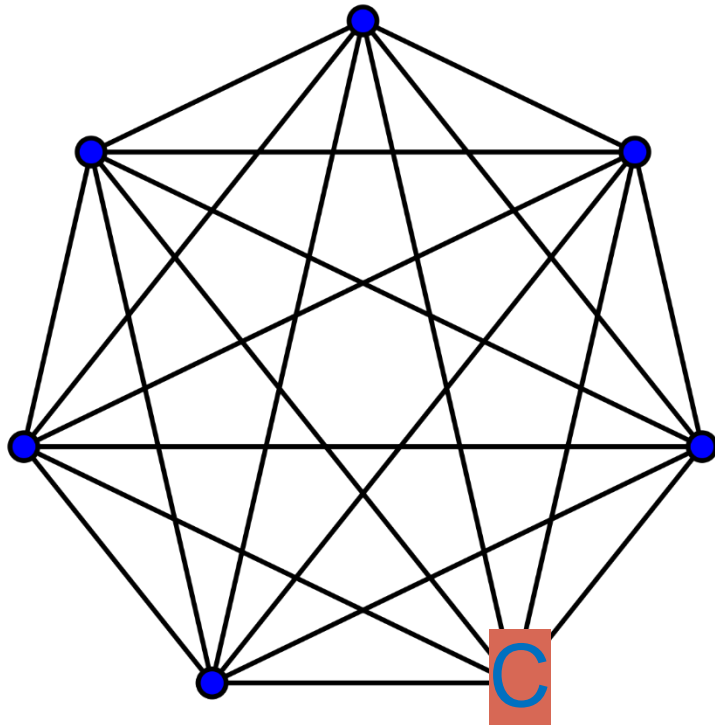


Cop-win graph

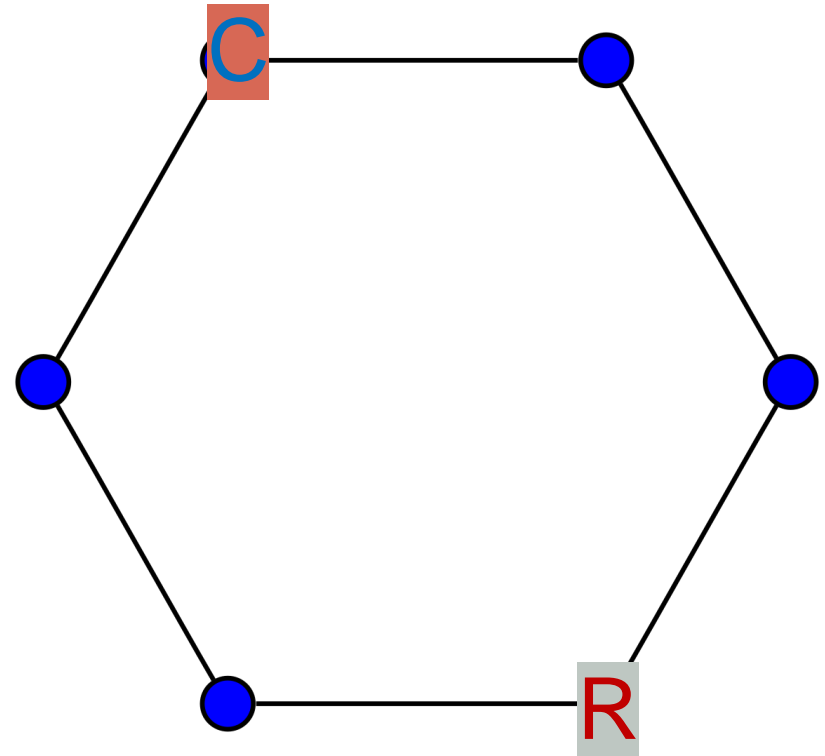


Rob-win graph

2-2. Type of Graphs

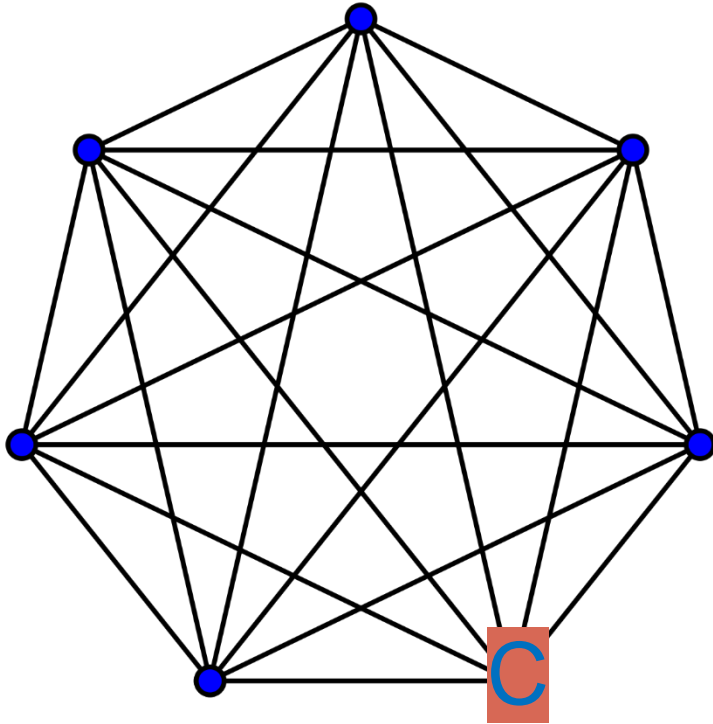


Cop-win graph

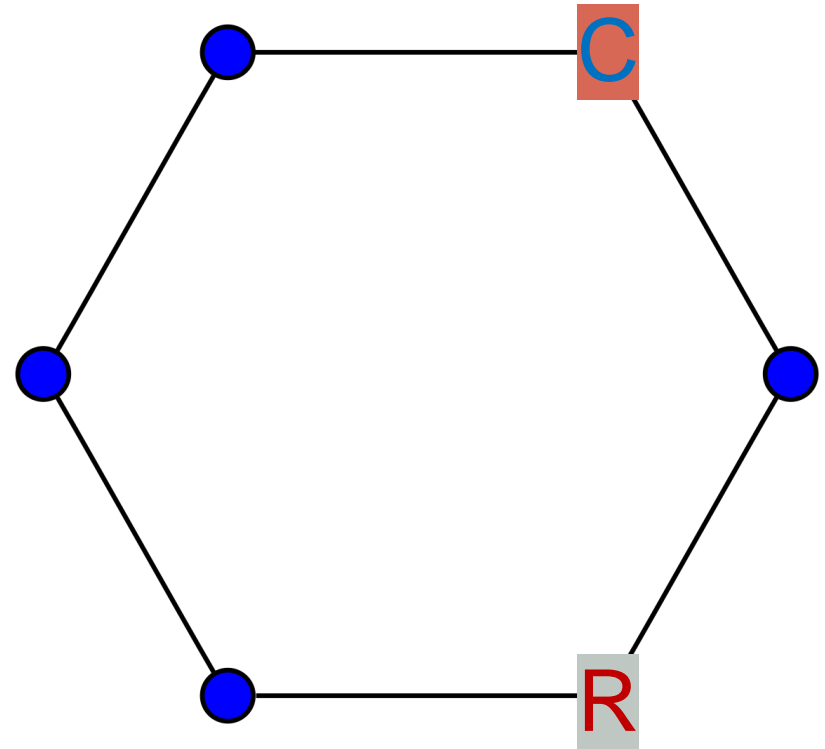


Rob-win graph

2-2. Type of Graphs

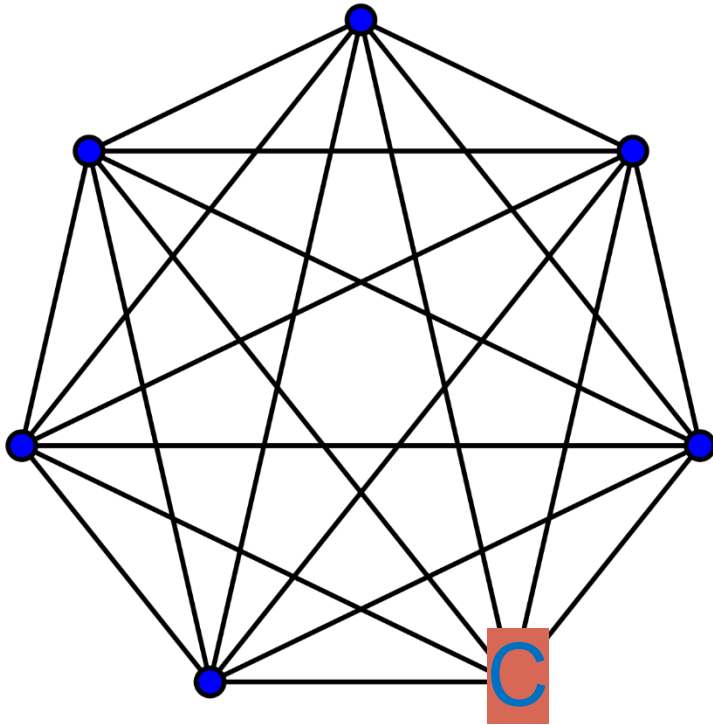


Cop-win graph

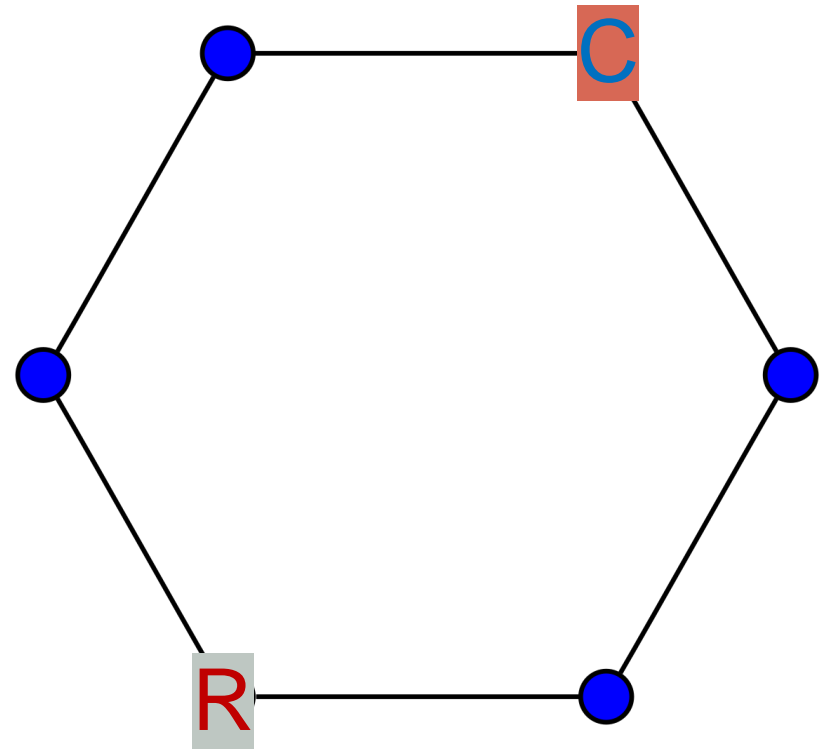


Rob-win graph

2-2. Type of Graphs



Cop-win graph

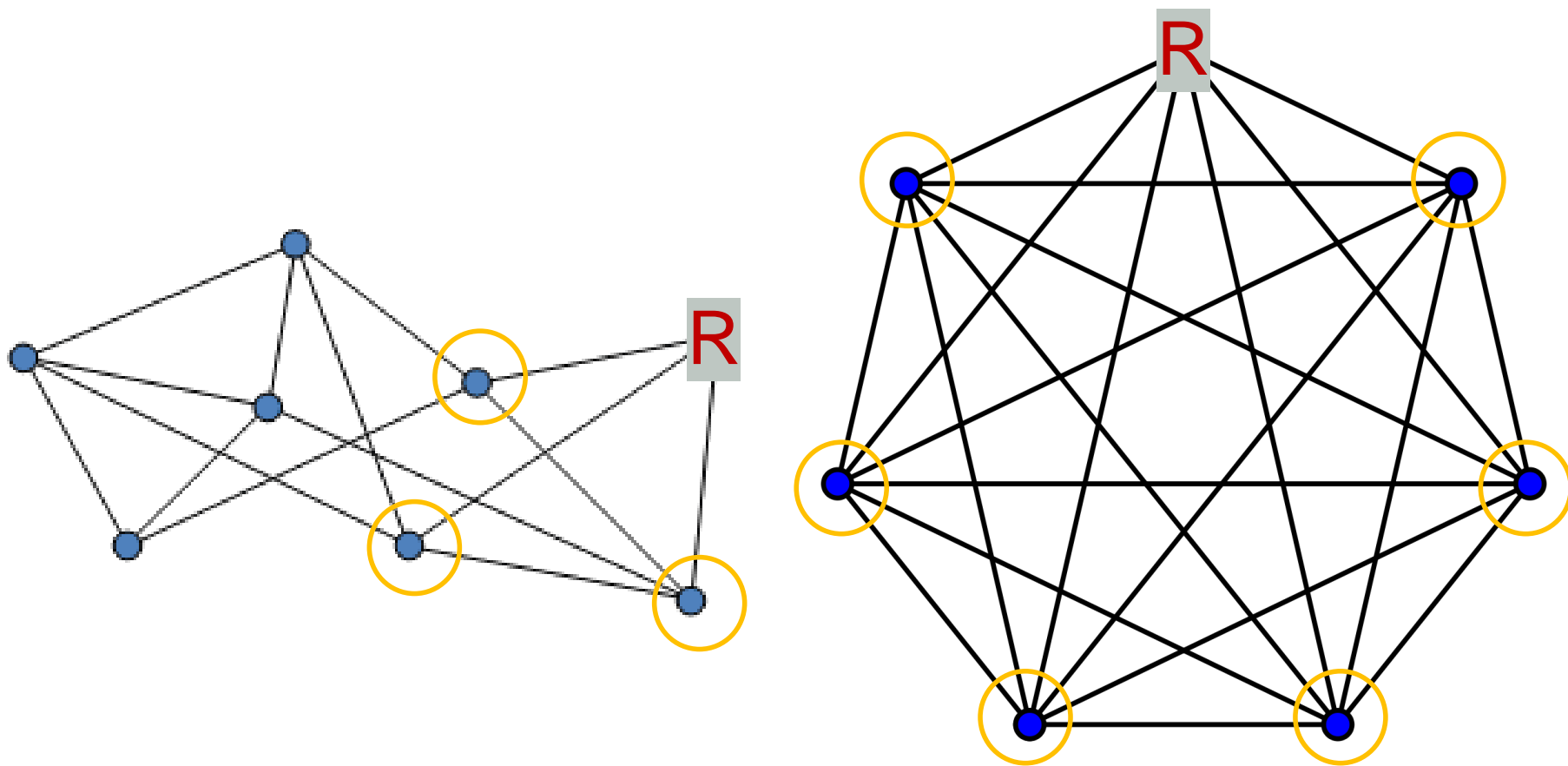


Rob-win graph

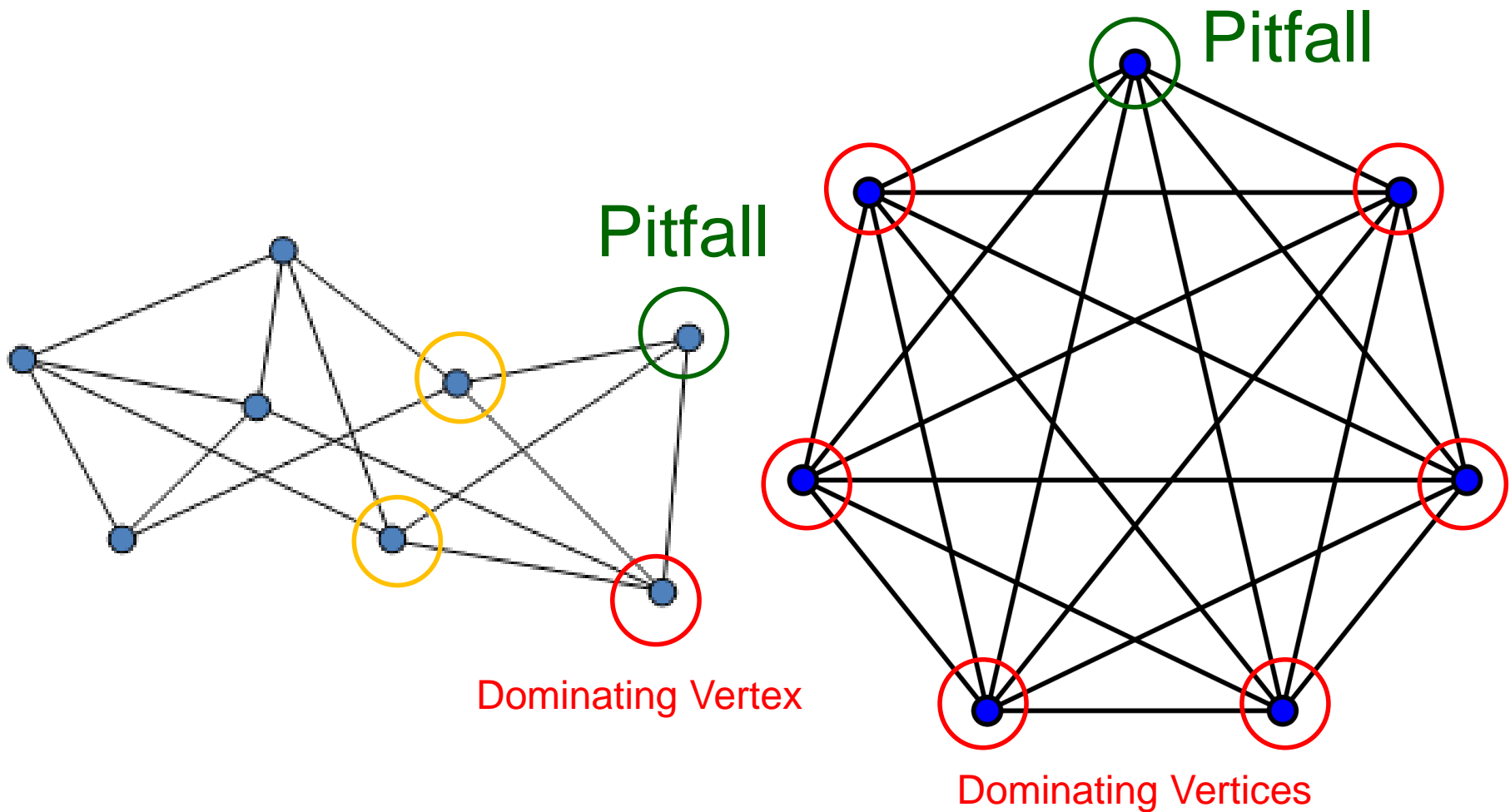
2-2. Type of Graphs

- N-Cop Win
 - N명의 Cop이 존재할 경우 항상 Cop이 승리
- $C(G)$
 - Graph G 에 대해서 N-Cop Win을 만족하는 최소의 N

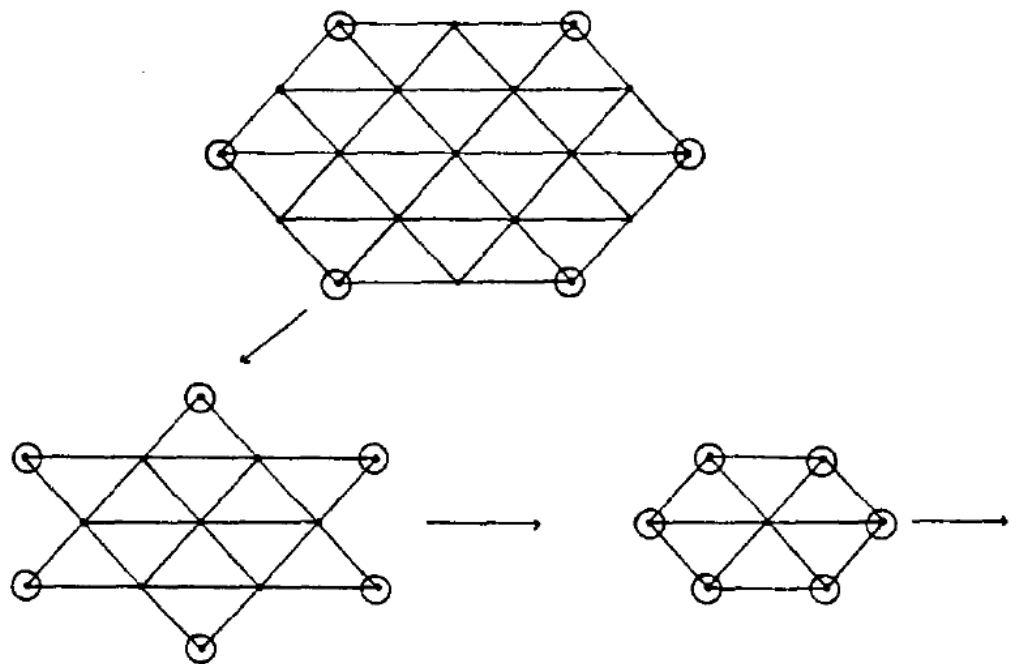
2-3. Pitfall & Dominating Vertex



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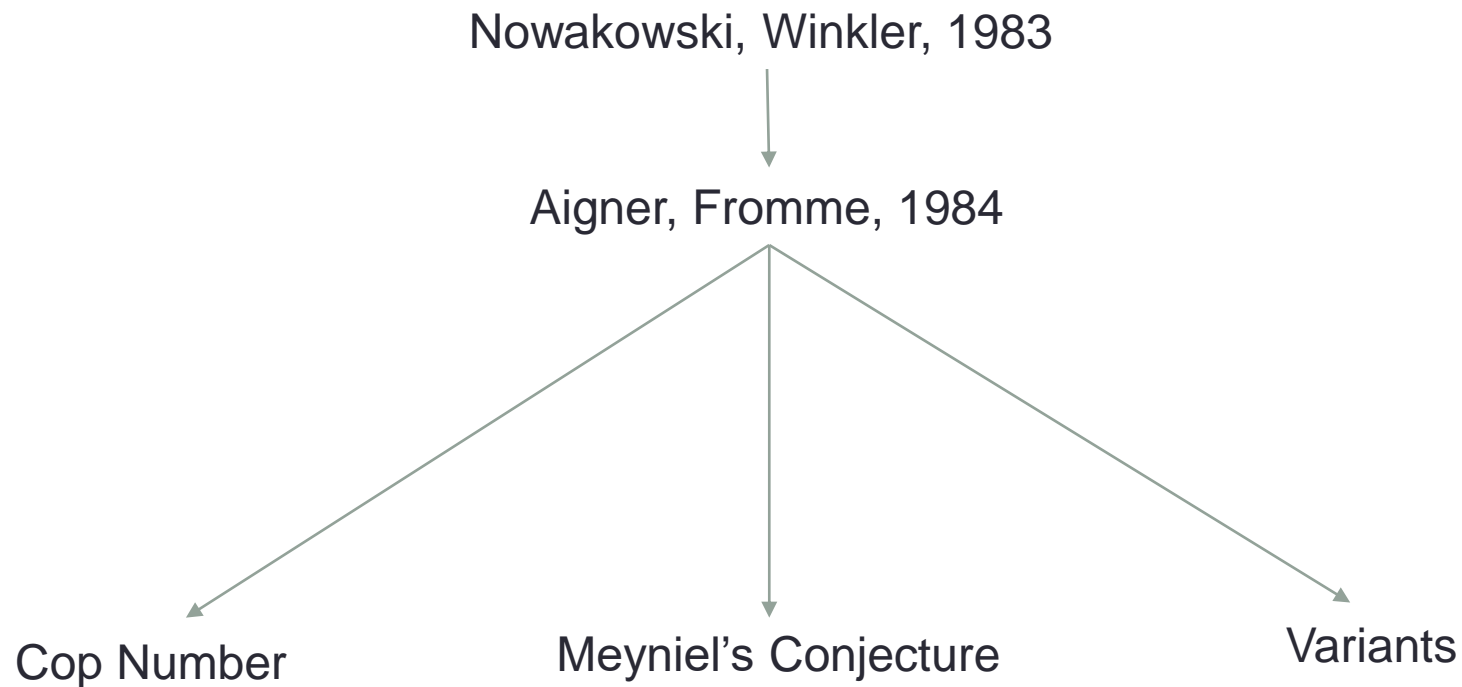


2-4. Aigner & Fromme's Theorem



- Pitfall의 제거 / 추가는 그래프의 결과(Cop-win, Rob-win)에 영향을 주지 않음
- 모든 Planar Graph에 대해서 $C(G) = 3$
 - Aigner, Fromme, 1984

3. Tree of C&R Problem



3-1. Cop Number Problem

- Directed Graph에서 Exp-complete
 - Goldstein & Reingold, 1995
- 최대 5 이하의 Degree를 가진 Maximal Planar Graph에 대해서 $C(G) = 2$
 - Maximal Planar Graph : Edge가 추가될 경우 Planar Graph가 아니게 되는 상태
 - Maurer, et al., 2010
- 모든 그래프에서 Exp-complete
 - Kinnersley, 2014

3-2. Meyniel's Conjecture

- 모든 n -node connected Graph에서 $C(G) = O(\sqrt{n})$
 - Meyniel, 1985

Meyniel's Conjecture [85]: For any n -node connected graph G , $cn(G) = O(\sqrt{n})$

	cn	
dominating set $\leq k$	$\leq k$	[folklore]
treewidth $\leq t$	$\leq t/2 + 1$	[Joret, Kaminski, Theis 09]
chordality $\leq k$	$< k$	[Kosowski, Li, Nisse, Suchan 12]
genus $\leq g$	$\leq \lfloor \frac{3g}{2} \rfloor + 3$	(conjecture $\leq g + 3$) [Schröder, 01]
H -minor free	$\leq E(H) $	[Andreae, 86]
degeneracy $\leq d$	$\leq d$	[Lu, Peng 12]
diameter 2	$O(\sqrt{n})$	—
bipartite diameter 3	$O(\sqrt{n})$	—
Erdős-Rényi graphs	$O(\sqrt{n})$	[Bollobas <i>et al.</i> 08] [Luczak, Pralat 10]
Power law	$O(\sqrt{n})$	[Bonato, Pralat, Wang 07]

A long story not finished yet...

- $cn(G) = O\left(\frac{n}{\log \log n}\right)$ [Frankl 1987]
- $cn(G) = O\left(\frac{n}{\log n}\right)$ [Chiniforooshan 2008]
- $cn(G) = O\left(\frac{n}{2^{(1-o(1))\sqrt{\log n}}}\right)$ [Scott, Sudakov 11, Lu, Peng 12]

note that $\frac{n}{2^{(1-o(1))\sqrt{\log n}}} \geq n^{1-\epsilon}$ for any $\epsilon > 0$

3-3. Variants

- Fast Robber

- 속도 s (양의 정수)로 Robber가 움직이는 경우
- Fomin, et al., 2010
- Fast robber의 분할그래프에서 NP-hard
- cliquewidth graph에서 강도 속도가 두 배일 경우 Polynomial
- Planar graph에서 경찰 수의 제한 X.

- Lazy Cop

- 한번에 한 명의 경찰만 이동가능
- Sullivan, et al., 2017

- Capture Time

- 3-cop win planar graph에 대해 추적 시간(Capture time) $\leq 2n$.
- Tan et al., 2017

- etc...